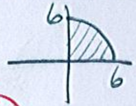
The time for a tax refund check to be deposited after it has been received is a continuous random variable X **SCORE: _____ / 10 PTS**

(in units of days) with probability density function $f(x) = \begin{cases} k\sqrt{36 - x^2}, & x \in [0, 6] \\ 0, & x \notin [0, 6] \end{cases}$ for some constant k.

Find the mean (average) time before a tax refund check is deposited.

$\int_0^6 k\sqrt{36-x^2}\,dx = 1$ $1\frac{1}{2}$



$k\left(\frac{1}{4}\cdot\pi(6)^2\right) = 1$

$k = \frac{1}{9\pi}$ $1\frac{1}{2}$

$1\frac{1}{2}$ $\int_0^6 \frac{1}{9\pi} x\sqrt{36-x^2}\,dx$

$u = 36 - x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$x = 6 \rightarrow u = 0$

$x = 0 \rightarrow u = 36$

3 $= \int_{36}^0 \frac{1}{9\pi}\left(-\frac{1}{2}\sqrt{u}\right)du$

1 $= -\frac{1}{\cancel{18}_9\pi}\,\frac{\cancel{2}}{3}\,u^{\frac{3}{2}}\Big|_{36}^{0}$

$= \frac{1}{27\pi}\cdot 36^{\frac{3}{2}}$

$= \frac{1}{27\pi}6^3$

$= \frac{8}{\pi}$ DAYS

$1\frac{1}{2}$

Find the length of the parametric curve $\begin{array}{l} x = 1 + 4t^{\frac{3}{2}} \\ y = \frac{3}{4}t^3 - 4\ln t \end{array}$ for $1 \le t \le e$.

SCORE: _____ / 6 PTS

$$\underbrace{\int_1^e \sqrt{(6t^{\frac{1}{2}})^2 + \left(\frac{9}{4}t^2 - \frac{4}{t}\right)^2}}_{2}\, dt$$

$$= \int_1^e \sqrt{\left(36t + \frac{81}{16}t^4 - 18t + \frac{16}{t^2}\right)}\, dt$$

$$= \int_1^e \underbrace{\sqrt{\frac{81}{16}t^4 + 18t + \frac{16}{t^2}}}_{1}\, dt$$

$$= \int_1^e \underbrace{\left(\frac{9}{4}t^2 + \frac{4}{t}\right)}_{1} dt$$

$$= \underbrace{\left(\frac{3}{4}t^3 + 4\ln t\right)}_{1}\Big|_1^e = \frac{3}{4}e^3 + 4 - \frac{3}{4} = \underbrace{\frac{3}{4}e^3 + \frac{13}{4}}_{1}$$

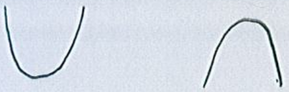
Find the center of mass of the region between the curves $y = 11x^2$ and $y = 12x - x^2$. **SCORE: _____ / 14 PTS**

$11x^2 = 12x - x^2$

$12x^2 - 12x = 0$

$12x(x-1) = 0$

$x = 0, 1$ 1



ON TOP

$\int_0^1 (12x - x^2 - 11x^2)\,dx = \int_0^1 (12x - 12x^2)\,dx = (6x^2 - 4x^3)\Big|_0^1 = 2$ 1 1½ 1

$\int_0^1 x(12x - 12x^2)\,dx = \int_0^1 (12x^2 - 12x^3)\,dx = (4x^3 - 3x^4)\Big|_0^1 = 1$ 1 1½ 1

$\frac{1}{2}\int_0^1 ((12x - x^2)^2 - (11x^2)^2)\,dx = \frac{1}{2}\int_0^1 (144x^2 - 24x^3 - 120x^4)\,dx$ 1½

$= \frac{1}{2}(48x^3 - 6x^4 - 24x^5)\Big|_0^1 = 9$ 1½ 1

$\frac{1}{2}(1, 9) = \left(\frac{1}{2}, \frac{9}{2}\right)$ 2